

# On a neutrino theory of matter

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Physically observable particles are assumed to result from an interaction between massless positively and negatively oriented 2-component Weyl neutrinos. A simple quantum mechanical analysis of a composite system of Weyl neutrinos of opposite orientations with a certain specific interaction shows that such a model can exhibit a 2-fold branching and defect in the total energy of the system, which could then be interpreted as formation of massive particles.

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## 1. Introduction

The standard model has had considerable success as a unified theory of all elementary particles. Together with Higgs mechanism it is able to explain the existence and masses of several new bosons.

Nevertheless, it cannot be considered as a complete and fully satisfactory theory of all elementary particles. It cannot, for example, explain intrinsically why the proton is about 1836 times heavier than the electron.

In the standard model the phenomenon of neutrino oscillation [1] requires that neutrinos have non-zero mass.

In 1957 Heisenberg [2],[3] tried to formulate (without much success) a unified theory of all elementary particles starting from a non-linear 4-component spinor equation with a built in fundamental constant.

In this paper we [4],[5] suggest that massless 2-component Weyl neutrinos, instead of 4-component spinors, are probably more fundamental than previously thought. We consider a composite system consisting of a massless positively oriented 2-component Weyl neutrino and a massless negatively oriented 2-component Weyl neutrino with a certain specific symmetry-breaking interaction between the two.

We assume that the observable physical particles manifest as energy states of the resulting 4-component system. A simple quantum mechanical treatment shows that such a model

should exhibit 2-fold branching and energy defects, which could then be interpreted as formation of particles of non-zero rest mass.

Such a model can also provide a qualitative, alternative non-standard explanation of the different flavors of a massless 4-component neutrino and thus of neutrino oscillation without assuming a neutrino mass.

The next step would be to consider such a model in the framework of quantum field theory.

## 2. Positively and negatively oriented 2-component Weyl neutrinos

The Weyl equation

$$(\sigma_k \partial_k + i \partial_4) \varphi = 0 \quad (1)$$

describes a massless positively oriented (i.e. left-handed) 2-component Weyl neutrino  $\nu_L$ . Here  $\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$ . We use space-time coordinates  $x_\mu (= x, y, z, ict)$ ,  $\mu = 1, 2, 3, 4$  and  $k = 1, 2, 3$ . The Pauli spin matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (2)$$

satisfy

$$\sigma_1 \sigma_2 = i \sigma_3, \quad \sigma_2 \sigma_3 = i \sigma_1, \quad \sigma_3 \sigma_1 = i \sigma_2, \quad (3)$$

Consider now the following equation

$$(\varrho_k \partial_k + i \partial_4) \chi = 0 \quad (4)$$

where the matrices  $\varrho_k$

$$\varrho_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \varrho_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \varrho_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (5)$$

differ from the Pauli spin matrices  $\sigma_k$  only in the interchange of the indices 1 and 3. The set  $\varrho_k$  satisfy

$$\varrho_1 \varrho_2 = -i \varrho_3, \quad \varrho_2 \varrho_3 = -i \varrho_1, \quad \varrho_3 \varrho_1 = -i \varrho_2, \quad (6)$$

Therefore, Eq.(4) describes a massless negatively oriented (i.e. right-handed) 2-component Weyl neutrino  $\nu_R$ .

## 3. Composite $\nu_L$ - $\nu_R$ system.

Consider first a composite  $\nu_L$ - $\nu_R$  system without interaction. The Hamiltonian of a positively oriented (massless) Weyl neutrino  $\nu_L$

described by (1) is given by  $H_L = -i\hbar(\sigma \cdot \nabla)$  with  $\sigma = (\sigma_1, \sigma_2, \sigma_3)$  and its eigenfunctions  $\varphi_{E_L}$  satisfy

$$\left. \begin{aligned} H_L \varphi_{E_L} &= E_L \varphi_{E_L} \\ \text{or } (\sigma \cdot \nabla) \varphi_{E_L} &= (iE_L/\hbar) \varphi_{E_L} \end{aligned} \right\} \quad (7)$$

From now on we shall adopt the conventional units in which  $c = \hbar = 1$ .

The solutions of (7) are well-known:

$$\left. \begin{aligned} \varphi_{E_L}(\mathbf{x}, \mathbf{p}) &= a(\mathbf{p}) e^{i\mathbf{x} \cdot \mathbf{p}} \\ \text{where } \mathbf{p}^2 \equiv p_1^2 + p_2^2 + p_3^2 &= E_L^2 \quad \text{and} \quad a(\mathbf{p}) = \begin{pmatrix} p_1 - ip_2 \\ E_L - p_3 \end{pmatrix} \end{aligned} \right\} \quad (8)$$

They describe a positively oriented Weyl neutrino  $\nu_L$  with energy  $E_L$ .

The spectrum of  $H_L$  is not discrete and hence the eigenfunctions have to be normalized by the delta-function.

$$\langle \varphi_{E_L}(\mathbf{x}, \mathbf{p}) | \varphi_{E'_L}(\mathbf{x}, \mathbf{p}') \rangle = a(\mathbf{p})^\dagger a(\mathbf{p}') \delta(\mathbf{p} - \mathbf{p}') \quad (9)$$

The eigenfunctions are also  $\infty$ -fold degenerate, since  $\mathbf{p}$  can take any value on the energy shell. We shall remove this degeneracy by integrating  $\varphi_{E_L}(\mathbf{x}, \mathbf{p})$  over the energy shell  $S_{E_L}^2 : p_1^2 + p_2^2 + p_3^2 = E_L^2$  which is a 2-sphere of radius  $E_L$ . We get (with a slight abuse of notation)

$$\varphi_{E_L}(\mathbf{x}) = \int_{S_{E_L}^2} a(\mathbf{p}) e^{i\mathbf{x} \cdot \mathbf{p}} dS_p \quad (10)$$

as a surface integral over  $S_{E_L}^2$ . Furthermore we shall suppose that  $\varphi_{E_L}(\mathbf{x})$  are normalised.

Similarly, the corresponding eigenfunctions for the negatively oriented Weyl neutrino  $\nu_R$  with Hamiltonian  $H_R = -i(\rho \cdot \nabla)$  with energy  $E_R$  are given by

$$\left. \begin{aligned} \chi_{E_R}(\mathbf{y}, \mathbf{q}) &= b(\mathbf{q}) e^{i\mathbf{y} \cdot \mathbf{q}} \\ \text{where } q_1^2 + q_2^2 + q_3^2 &= E_R^2 \quad \text{and} \quad b(\mathbf{q}) = \begin{pmatrix} q_3 - iq_2 \\ E_R - q_1 \end{pmatrix} \end{aligned} \right\} \quad (11)$$

We use  $\mathbf{y}$  for the coordinate of  $\nu_R$  and note that  $b(\mathbf{q})$  differs from  $a(\mathbf{p})$  by an interchange of the indices 1 and 3. A similar integration over the energy shell  $S_{E_R}^2 : q_1^2 + q_2^2 + q_3^2 = E_R^2$  gives

$$\chi_{E_R}(\mathbf{y}) = \int_{S_{E_R}^2} b(\mathbf{q}) e^{i\mathbf{y} \cdot \mathbf{q}} dS_q \quad (12)$$

Let  $\mathcal{H}_L$  and  $\mathcal{H}_R$  be the respective Hilbert spaces for  $\nu_L$  and  $\nu_R$ . Consider now a composite  $\nu_L$ - $\nu_R$  system without interaction given by the tensor product  $\mathcal{H} = \mathcal{H}_L \otimes \mathcal{H}_R$  with the Hamiltonian  $H_o = H_L + H_R$  (or more precisely  $H_L \otimes I + I \otimes H_R$ ). Since

$H_o = -i(\sigma \cdot \nabla_+) - i(\rho \cdot \nabla_-)$ , where  $\nabla_+, \nabla_-$  act on  $\mathbf{x}, \mathbf{y}$  respectively, and thus does not depend on  $\mathbf{x}$  and  $\mathbf{y}$  explicitly, not only

$$\Psi_1(\mathbf{x}, \mathbf{y}) = \varphi_{E_L}(\mathbf{x}) \otimes \chi_{E_R}(\mathbf{y}) \quad (13)$$

but also

$$\Psi_2(\mathbf{x}, \mathbf{y}) = \varphi_{E_R}(\mathbf{x}) \otimes \chi_{E_L}(\mathbf{y}) \quad (14)$$

are both eigenfunctions of  $H_o$  with energy  $E_L + E_R$ . Thus the composite system  $\nu_L - \nu_R$  behaves like a system of identical particles even though  $\nu_L$  is not identical to  $\nu_R$ .

*We remark that if the universe were spatially non-orientable,  $\nu_L$  would be indistinguishable from  $\nu_R$ . But in an orientable universe they would be distinct particles.*

#### 4. Composite $\nu_L - \nu_R$ system with interaction

Consider now an interaction  $V(\mathbf{x}, \mathbf{y})$  between  $\nu_L$  and  $\nu_R$  of the form

$$V(\mathbf{x}, \mathbf{y}) = F(|\mathbf{x}|)H(|\mathbf{x}| - |\mathbf{y}|) + F(|\mathbf{y}|)H(|\mathbf{y}| - |\mathbf{x}|) \quad (15)$$

where  $F$  is a function to be specified and  $H$  is the Heaviside function. Thus

$$V(\mathbf{x}, \mathbf{y}) = \left. \begin{array}{ll} F(|\mathbf{x}|) & \text{if } |\mathbf{x}| > |\mathbf{y}| \\ F(|\mathbf{y}|) & \text{if } |\mathbf{x}| < |\mathbf{y}| \end{array} \right\} \quad (16)$$

Note that  $V(\mathbf{x}, \mathbf{y}) = V(\mathbf{y}, \mathbf{x})$ . The combined Hamiltonian  $H = H_o + V(\mathbf{x}, \mathbf{y})$  with total energy say  $E_{Tot}$  satisfies an equation of the form:

$$[H_L + H_R + V(\mathbf{x}, \mathbf{y})]\Psi = E_{Tot}\Psi \quad (17)$$

We can now follow an approximation (i.e. perturbation) procedure similar to that of the He-atom [6]. (Although it would be perhaps more appropriate to use the Lippmann-Schwinger equation for the continuous case). Thus as a first approximation the total energy is given by

$$E_{Tot} = E_L + E_R + (J \pm K) \quad (18)$$

where

$$J = \langle \Psi_1 | V(\mathbf{x}, \mathbf{y}) | \Psi_1 \rangle = \langle \varphi_{E_L}(\mathbf{x}) \otimes \chi_{E_R}(\mathbf{y}) | V(\mathbf{x}, \mathbf{y}) | \varphi_{E_L}(\mathbf{x}) \otimes \chi_{E_R}(\mathbf{y}) \rangle \quad (19)$$

$$K = \langle \Psi_2 | V(\mathbf{x}, \mathbf{y}) | \Psi_1 \rangle = \langle \varphi_{E_R}(\mathbf{x}) \otimes \chi_{E_L}(\mathbf{y}) | V(\mathbf{x}, \mathbf{y}) | \varphi_{E_L}(\mathbf{x}) \otimes \chi_{E_R}(\mathbf{y}) \rangle \quad (20)$$

We shall now suppose that  $V$  is such that  $J$  is non-positive, i.e.  $J(E_L, E_R) = -\Gamma(E_L, E_R)$ , where  $\Gamma(E_L, E_R) \geq 0$  for  $E_L, E_R \geq 0$  (see example later), so that

$$E_{Tot} = E_L + E_R - (\Gamma \pm |K|) \quad (21)$$

The total energy of such an interactive system thus has two branches. Note that both  $\Gamma$  and  $|K|$  are functions of  $E_L$  and  $E_R$ . If we start with a  $\nu_L$  of energy  $E_L > 0$  and a  $\nu_R$

of energy  $E_R > 0$  and switch on the interaction, the total energy  $E_{Tot}$  would be positive only if  $E_L + E_R > \Gamma \pm |K|$ . The value of  $E_L + E_R$  where  $E_{Tot} = 0$  and changes sign, can be interpreted as the rest-mass of a free (stable) particle.

### 5.A reduced 1-dimensional model

In order to evaluate  $J$  and  $K$  for any specific model we need first to evaluate the surface integrals (10) and (12) over the energy shells  $S_{E_L}^2$  and  $S_{E_R}^2$ .

Setting  $p_3 = \pm \sqrt{E_L^2 - (p_1^2 + p_2^2)}$  and introducing the polar coordinates  $(r, \theta)$  in the  $p_1$ - $p_2$  plane, we get  $\varphi_{E_L}(\mathbf{x}) = \begin{pmatrix} \varphi_{E_L}^1(\mathbf{x}) \\ \varphi_{E_L}^2(\mathbf{x}) \end{pmatrix}$ , where

$$\varphi_{E_L}^1(\mathbf{x}) = \int_0^{E_L} \left( \int_0^{2\pi} e^{i(r a(\theta) - \theta)} d\theta \right) 2 \frac{\cos(x_3 \sqrt{E_L^2 - r^2}) E_L r^2}{\sqrt{E_L^2 - r^2}} dr \quad (22a)$$

$$\varphi_{E_L}^2(\mathbf{x}) = \int_0^{E_L} \left( \int_0^{2\pi} e^{i r a(\theta)} d\theta \right) \times \left. \begin{aligned} & 2 \frac{\left( E_L \cos(x_3 \sqrt{E_L^2 - r^2}) - i \sqrt{E_L^2 - r^2} \sin(x_3 \sqrt{E_L^2 - r^2}) \right) E_L r}{\sqrt{E_L^2 - r^2}} dr \end{aligned} \right\} \quad (22b)$$

Here  $a(\theta) = x_1 \cos(\theta) + x_2 \sin(\theta)$ .

Unfortunately, the above integrals cannot be evaluated in closed form. Consider therefore a reduced one-dimensional model with  $x_1 = x_2 = 0$ . Then  $a(\theta) = 0$ , so that (setting  $x_3 = x$ )

$$\varphi_{E_L}(x) = \begin{pmatrix} 0 \\ 4 \frac{(E_L \sin(E_L x) x - i \sin(E_L x) + i x E_L \cos(E_L x)) E_L \pi}{x^2} \end{pmatrix} \quad (23)$$

$\varphi_{E_L}(x)$  is square integrable, that is

$$\langle \varphi_{E_L}(x) | \varphi_{E_L}(x) \rangle = \int_{-\infty}^{\infty} \varphi_{E_L}(x)^\dagger \varphi_{E_L}(x) dx = \left(\frac{64}{3}\right) \pi^3 E_L^5 \quad (24)$$

The normalizing factor for  $\varphi_{E_L}(x)$  is thus  $N_{E_L} = \left(\left(\frac{64}{3}\right) \pi^3 E_L^5\right)^{-1/2}$ . From now on we shall suppose that  $\varphi_{E_L}(x)$  is normalised.

Similarly, setting  $y_1 = y_2 = 0, y_3 = y$  we get from (12)

$$\chi_{E_R}(y) = \begin{pmatrix} \frac{4 i (\sin(E_R y) - E_R y \cos(E_R y)) \pi E_R}{y^2} \\ 4 \frac{\sin(E_R y) \pi E_R^2}{y} \end{pmatrix} \quad (25)$$

and

$$\langle \chi_{E_R}(y) | \chi_{E_R}(y) \rangle = \int_{-\infty}^{\infty} \chi_{E_R}(y)^\dagger \chi_{E_R}(y) dy = \left(\frac{64}{3}\right) \pi^3 E_R^5 \quad (26)$$

so that the normalising factor for  $\chi_{E_R}(y)$  is also  $N_{E_R} = ((\frac{64}{3})\pi^3 E_R^5)^{-1/2}$ . Again suppose that from now on  $\chi_{E_R}(y)$  is normalised.

According to (19)

$$J = \int \int \varphi_{E_L}(x) \dagger \varphi_{E_L}(x) V(x, y) \chi_{E_R}(y) \dagger \chi_{E_R}(y) dx dy \quad (27)$$

Using a mean-value theorem for integrals we can write

$$J = \varphi_{E_L}(\xi) \dagger \varphi_{E_L}(\xi) \chi_{E_R}(\eta) \dagger \chi_{E_R}(\eta) \int \int V(x, y) dx dy \quad (28)$$

where  $-\infty < \xi, \eta < \infty$ .

Now *assume* that  $V$  is given by (16), where the function  $F$  is, for  $x > 0$  :

$$F(x) = e^{-\mu x} \ln(x) \quad (29)$$

Here  $\mu > 0$  is some fundamental interaction constant.

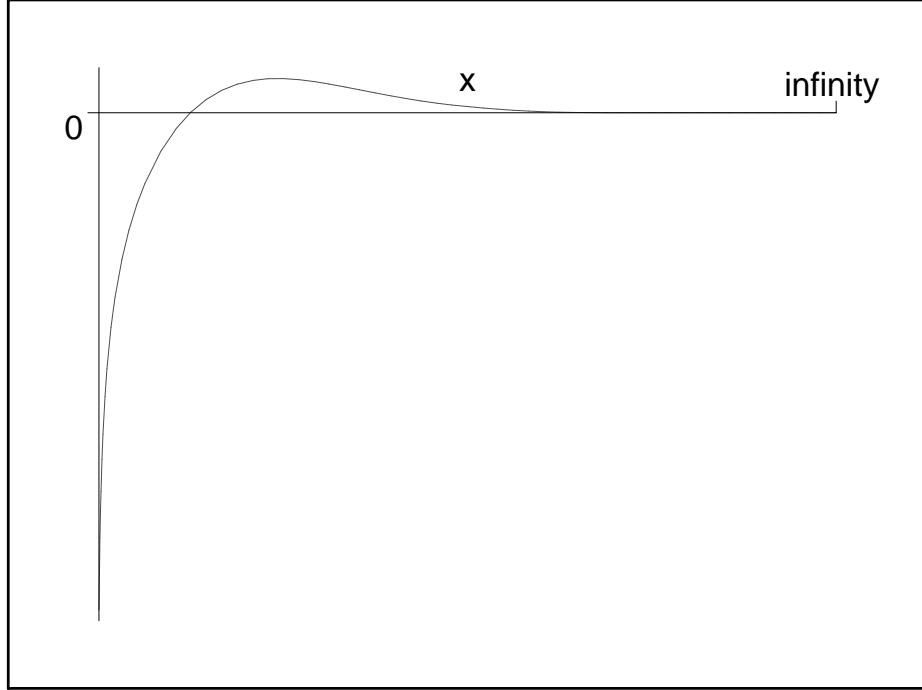


Fig.1

Fig.1 shows  $F(x)$  as a function of  $x$ .

Then

$$\left. \begin{aligned} \int \int V(x, y) dx dy &= 4 \int_0^\infty dy \left[ \int_0^y F(y) dx + \int_y^\infty F(x) dx \right] \\ &= -8 (\ln(\mu) + \gamma - 1) / \mu^2 \equiv -\Gamma_o \end{aligned} \right\} \quad (30)$$

Here  $\gamma = 0.5772156649..$  (Euler constant). Hence  $\Gamma_o > 0$  if  $\mu > e^{1-\gamma} = 1.526205112..$ . We shall suppose that this condition is satisfied, so that  $J = -\Gamma \leq 0$ , where

$$\Gamma(E_L, E_R) = \varphi_{E_L}(\xi) \dagger \varphi_{E_L}(\xi) \chi_{E_R}(\eta) \dagger \chi_{E_R}(\eta) \Gamma_o \quad (31)$$

Similarly

$$K(E_L, E_R) = -\varphi_{E_R}(\vartheta) \dagger \varphi_{E_L}(\vartheta) \chi_{E_L}(\zeta) \dagger \chi_{E_R}(\zeta) \Gamma_o \quad (32)$$

where  $-\infty < \vartheta, \zeta < \infty$ .

In order to obtain a qualitative idea of how the two branches of  $E_{Tot}$  behave as a function of  $E_L$  and  $E_R$ , consider the case where  $E_L = E_R = E$ . Then  $\Gamma = |K|$  and for the branch  $E_{Tot} = 2E - (\Gamma + |K|)$  we have, from (23), (25) and (31)

$$E_{Tot} = 2E - 3/4 \left. \begin{aligned} & \frac{E^2 \xi^2 + 1 - (\cos(E\xi))^2 - 2 \sin(E\xi) \xi E \cos(E\xi)}{\pi E^3 \xi^4} \times \\ & 3/4 \frac{E^2 \eta^2 + 1 - (\cos(E\eta))^2 - 2 \sin(E\eta) \eta E \cos(E\eta)}{\pi E^3 \eta^4} \Gamma_o \end{aligned} \right\} \quad (33)$$

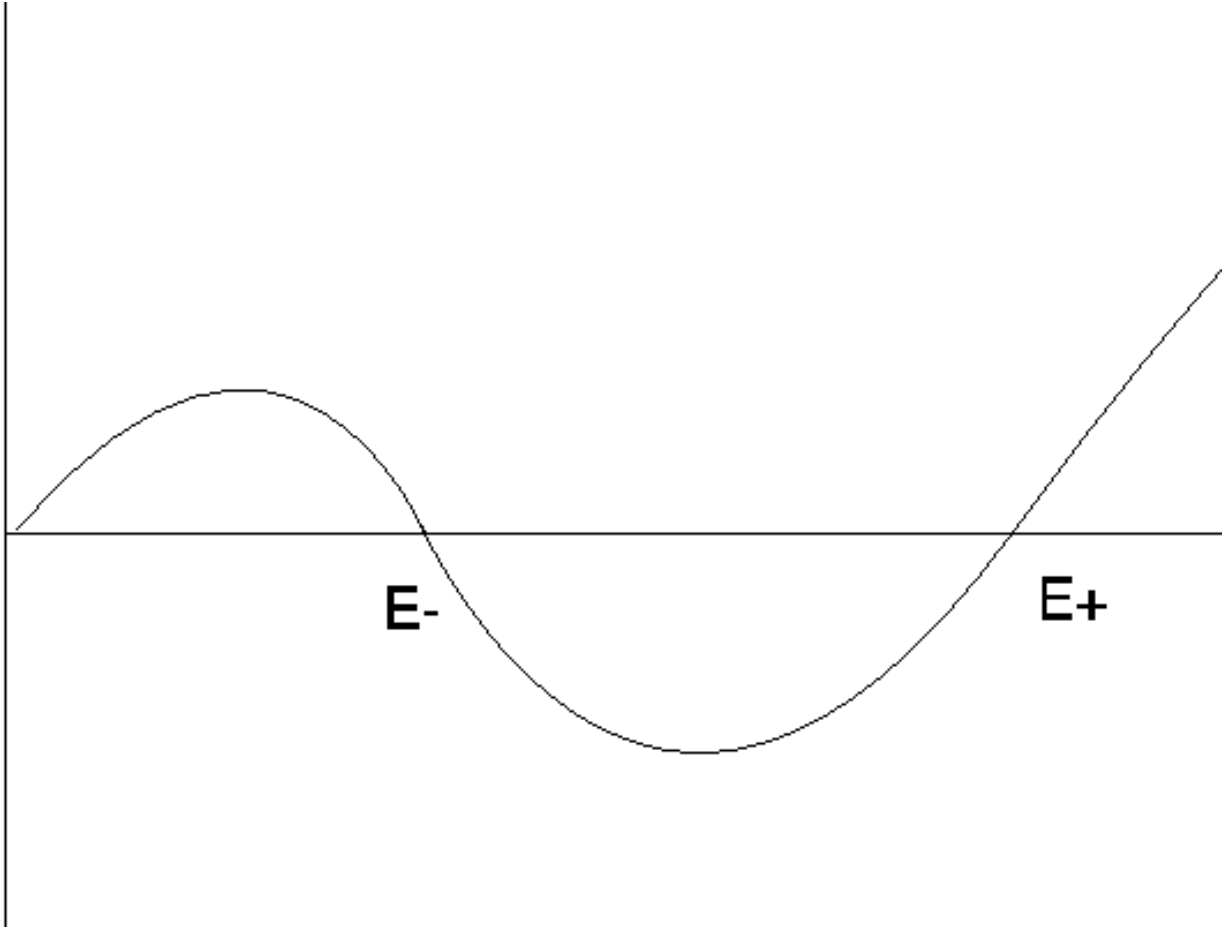


Fig.2  $E_{Tot}$  as a function of  $E$

Fig.2 is a plot of  $E_{Tot}$  as a function of  $E$  for some specific values of  $\xi, \eta, \Gamma_o$ . It shows that, in general, for values  $0 < E < E-$ ,  $E_{Tot}$  is positive; it becomes zero at  $E = E-$ ; it is then negative when  $E- < E < E+$ . It is positive again when  $E > E+$ .

How can one interpret this somewhat strange behaviour of  $E_{Tot}$  as a function of  $E$ ?

A possible interpretation is as follows.

Now imagine that we start with a free  $\nu_L$  and a free  $\nu_R$  both of energy  $E$ . The free system has a mirror symmetry. If we now switch on the above mentioned interaction, the symmetry is broken because of the nature of the Heaviside function in the interaction. Then depending on if  $0 < E < E-$  the composite system is a 4-component Dirac neutrino of energy  $E_{Tot} > 0$ . If however  $E- < E < E+$  the composite system would contribute to vacuum energy  $E_{Tot} < 0$ . If  $E > E+$  we have a positive energy defect of  $2E+$ . This can be interpreted as the formation of a particle of rest mass equal to  $2E+$  and  $E_{Tot}$  now representing the pure kinetic energy of the formed particle.

The rest mass  $2E+$  would depend on the value of the fundamental constant  $\mu$ .



A similar analysis of the other branch (with  $E_L \neq E_R$ ) would lead to another particle with rest mass less than the above case.

## 6. Conclusion

We have thus shown that a simple quantum mechanical analysis of a composite  $\nu_L$ - $\nu_R$  model with a specific symmetry-breaking interaction suggests a possible formation of particles of non-zero rest mass from 2-component Weyl neutrinos of sufficiently high energy. In this interactive model, in a model universe filled with Weyl neutrinos  $\nu_L$  and  $\nu_R$  with energy spectrum  $0 < E_L, E_R < \infty$ , we can therefore expect *at least* three things: 4-component Dirac neutrinos, a vacuum filled with negative energy and two kinds of stable particles of non-zero rest mass.

The analysis presented here is in the framework of quantum mechanics of particles. The next step would be to consider such a model in the framework of quantum field theory.

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